A neural circuit model of decision making

Xiao-Jing Wang
Department of Neurobiology & Kavli Institute for Neuroscience
Yale University School of Medicine
Three basic questions on decision computations

- How does a neural circuit accumulate information to gather evidence over time? How is a categorical choice produced, and what is a decision threshold in neuronal terms?
- Is there a common mechanism for perceptual decisions and value-based choice behavior? What is the source of stochasticity in probabilistic decision making?
- Can Bayes-like inference be realized by simple neural mechanisms in the brain? Does decision behavior generally deviate from Bayes-optimality?
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M. I. Posner (1978)
Chronometric explorations of mind

R. D. Luce (1986)
Response times: their role in inferring elementary mental organization
Probabilistic decision-making in a visual motion two-alternative forced choice task
Roitman and Shadlen 2002
PFC

Working memory & decision making
MT

Motion strength (% coherence)

Britten et al. 1996
Delayed response task

Shadlen and Newsome J Neurophysiol 2001
Reaction Time Task

Roitman and Shadlen 2002
Ramping-to-threshold (drift diffusion) Model

\[ \frac{dR}{dt} = I_A - I_B + \text{noise}, \quad R(t) = (I_A - I_B)t + \int_0^t dt \text{ noise}. \]

R Ratcliff (Psychol Rev 1978, Neural Comput 2007)
J Schall (Nature Rev Neurosci 2001)
Mazurek et al (Cereb Cortex 2003)
Membrane Time Constant = 10-20 ms
Lorente de Nó’s reverberatory circuit

- Levitt et al 1993
- Kritzer and Goldman-Rakic 1995
- Pucak et al 1996
Recurrent excitation prolongs integration time

\[
\frac{dr}{dt} = \frac{-r}{\tau_{syn}} + I
\]

\[
\tau_{network} = \frac{\tau_{syn}}{1 - w_{rec}}
\]

If \( w_{rec} = 0.9 \), \( \tau_{syn} = 100 \text{ms} \), \( \tau_{network} = 1 \text{ sec} \)

If \( w_{rec} = 1 \) (fine-tuning!) then \( \tau_{network} = \infty \)

\( \rightarrow \) Perfect integrator
A simple cortical circuit model for decision making and working memory

- 2-population (excitatory and inhibitory) spiking neurons
- Slow reverberatory excitation mediated by the NMDA receptors at recurrent synapses
- Winner-take-all competition by feedback inhibition

Motion coherence
\[ c' = \frac{I_A - I_B}{I_A + I_B} \]
Quantitative differences give rise to qualitatively different behaviors.

![Graph showing firing rate vs. synaptic strength](image-url)
Spontaneous symmetry breaking and stochastic decision making

![Graph showing firing rate vs. time and firing rate distributions for two trials.](image)
Irregular spike firing of cortical cells:
CV (SD/mean of inter-spike intervals)
≈0 for a regular oscillator, ≈ 1 for a Poisson process

Spiking activity of neurons in monkey prefrontal cortex
(Compte et al J Neurophysiol 2003)
Memory of a choice during delay

Stimulus with $c' = 6.4\%$

resting state
Model

![Graph showing coherence index vs. % correct for reaction time and fixed duration]

Monkey Data

![Graph showing motion strength vs. % correct for reaction time and fixed duration]
Neural substrate of a decision threshold?

A bursty neuron in superior colliculus

Fixation
Target

D Munoz and R Wurtz
A Large-Scale Network Model of Decision-Making

Lo and Wang
Performance improves with longer time integration, but shows a plateau effect.

Wang, Neuron 2002
Kiani et al 2008
Accumulation of time-varying evidence

Wong, Huk, Shalden and Wang 2007
Huk and Shadlen 2005
Inhibitory Control of Action

Countermanding Task

Schall Nat Rev Neurosci 2001
Stochastic network dynamics *prior to stop signal* largely determines whether the planned action will be cancelled or not.

Lo, Boucher, Paré, Schall and Wang, J Neurosci 2009
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SENSORY INPUT

- low level sensory analyzers

DECISION MECHANISMS

- motor output structures

ADAPTIVE BEHAVIOR

representation of stimulus/action value

W Newsome
The neural basis of economic choice behavior: Decision circuit endowed with reward-dependent learning

Stochastic Hebbian Learning with Binary Synapses

- A fraction $c$ of synapses are in the potentiated state (with strength $G_{\text{up}}$) and the remaining synapses are in the depotentiated state (strength $G_{\text{down}}$). Amit and Fusi 1992, 1994; Petersen et al. 1998, O’Connor et al 2005

- Synaptic changes are Hebbian, e.g. $r_{\text{pre}}=$high; LTP if $r_{\text{post}}=$high and LTD if $r_{\text{post}}=$low

- LTP or LTD occurs stochastically, with transition probability $q_+$ or $q_-$, respectively. Amit and Fusi 1992, 1994; Fusi 2002

- Plasticity is gated by reward (e.g. dopamine may reverse the sign of plasticity) (Reynolds and Wickens 2000)
Choice probability depends on the difference between synaptic strengths.

\[ p_A = \frac{1}{1 + \exp\left(-\frac{(c_A - c_B)}{\sigma}\right)} \]

Synaptic modification takes place only for the winner population.

\[
\begin{align*}
A \text{ selected and rewarded} & : \begin{cases} \\
\Delta c_A = (1 - c_A)q_+ \\
\Delta c_B = 0 \\
\end{cases} \\
A \text{ selected but not rewarded} & : \begin{cases} \\
\Delta c_A = -c_Aq_- \\
\Delta c_B = 0 \\
\end{cases}
\]

Soltani & Wang, J Neurosci 2006
Dynamic foraging task & Herrnstien’s Matching Law

Adapted from Sugrue et al. 2004

Sugrue et al 2004, Lau and Glimcher 2005
Synaptic strengths learn returns

\[ \Delta c_i = q_+(1 - c_i)I_i - q_- c_i(P_i - I_i). \]

\[ c_i^{ss} = \frac{q_+ I_i}{(q_+ - q_-)I_i + q_- P_i} = \frac{q_+ R_i}{(q_+ - q_-)R_i + q_-} \]

Choice probability \( P_i = \frac{\text{(# trials choosing i)}}{\text{(total trial #)}} \)
Income \( I_i = \frac{\text{(# rewards on target i)}}{\text{(total trial #)}} \)
Return \( R_i = \frac{I_i}{P_i} = \frac{\text{(# rewards on target i)}}{\text{(# trials choosing i)}} \)
\[ q_+ = q_- \]

\[ q_+ = 2q_- \]

\[ 2q_+ = q_- \]

\[ \text{Return}_i = \text{reward}_i / \text{choice}_i, \ i = A \text{ or } B \]
Model simulation of matching behavior

Blue: choice ratio
Why undermatching?

\[ I_{\text{tot}} = P_A R_A + P_B R_B \]
Interactive Game with an Opponent

Alireza Soltani, Daeyeol Lee
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Weather Forecasting

\[ WOE = \log_{10} \frac{P(\text{red})}{P(\text{green})} \]

Example:

\[ P(\text{red}) = \frac{10 \sum WOE}{1 + 10 \sum WOE} \]

Yang and Shadlen, Nature 2007
Model of Weather Prediction Task
Soltani and Wang, Nature Neurosci 2009

\[
p_A = \frac{1}{1 + \exp\left(-\sum_{s_i \in t_k} \frac{(c_{iA} - c_{iB})}{\sigma}\right)}
\]

\[
\log \frac{p_A}{1 - p_A} = \frac{1}{\sigma} \sum_{s_i \in t_k} (c_{iA} - c_{iB})
\]
\[
\log \frac{p_A}{1 - p_A} = \frac{1}{\sigma} \sum_{s_i \in t_k} (c_i A - c_i B)
\]

\[
c_A \approx P(A|s), \quad c_B \approx P(B|s)
\]

\[
\log \frac{p_A}{1 - p_A} \propto \sum_{s_i \in t_k} P(A|s_i) - P(B|s_i)
\]

\[
\log \frac{p_A}{1 - p_A} \propto \sum_{s_i \in t_k} \log_{10} \left( \frac{P(A|s_i)}{P(B|s_i)} \right)
\]

\[
P(A|s_i) - P(B|s_i) \simeq \log_{10} \left( \frac{P(A|s_i)}{P(B|s_i)} \right)
\]

\[
2x - 1 \simeq \log_{10} \left( \frac{x}{1-x} \right)
\]

\[
0.2 \leq x \leq 0.8
\]
Synapses combine Log LR with prior to encode posterior probability
But choice behavior is biased by prior

\[
\sum_i \Delta c_i^{ss} = \sum_i \left[ \alpha \log_{10} \left( \frac{\tilde{P}(A|S_i)}{\tilde{P}(B|S_i)} \right) + \beta \log_{10} \left( \frac{P(A)}{P(B)} \right) \right]
\]

\( \beta < 0 \)

Choice bias by pattern
Choice bias by a single cue

Base-rate neglect?
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